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Reduced Order Model Based on Dominant and Non-Dominant State Of Discrete Time Control Systems

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ABSTRACT:

Analysis and implementation of large scale linear control systems are both tedious and costly and too complicated to be used in real problems.Therefore, to analyze such large scale linear control systems, it is necessary to reduce it to a lower order system, which is a sufficient representation of the higher order system.In this paper,we propose a model order reduction of large scale linear control systems. First, identify the dominant and non-dominant states for alinear system using modal approach.Develop a reduced order model for reduction of large-scale linear discrete-time control system using dominant and non-dominant state of a system. The reduced order model is used for control applications like reduced order controller, reduced order observer and observer-based reduced order controller for the linear control system. Lyapunov stability theory is the methodology used for establishing the reduced order controllers and observers for the large-scale discrete-time linear systems.Reduced order control is used to stabilize the system and observers are used to determine the system is observable or not.Observer-based and functional observer strategies for the large scale discrete-time linear control systems will be investigated, which shall enhance their performance in industrial applications. Reduced order functional observers will also be derived for large scale linear control systems, which have important applications in implementing controllers for the linear systems.

KEYWORDS: Reduced Order model, Control systems, Observer,Linear system

INTRODUCTION:

There is an incredible request in the control frameworks writing for acknowledging low request straightforward direct models for the first extensive scale straight control systems.Deriving lessened request models for vast scale direct frameworks has been a dynamic region of research in the control frameworks writing. The utilization of a decreased request display makes it less demanding to actualize investigation, reenactments and control framework outlines. Additionally, there is an incredible request from the business to utilize low request models instead of high request models for physical or mechanical frameworks since it is anything but difficult to work with low request models and outline controllers, spectators, and so forth.,

From 1960s, there has been dynamic research in the control writing for determining lessened request models for substantial scale straight frameworks models, which can be grouped into two wide composes, viz. state-space outline strategies (present day control techniques) and exchange work strategies (established control strategies).

The principle destinations of the proposed framework are as per the following:

(1) Identification of predominant and non-overwhelming conditions of huge scale direct discrete-time control frameworks

(2) Reduced request display plan for the straight discrete-time control frameworks

(3) Reduced request controllers and onlookers for huge scale discrete-time direct frameworks.

(4) Reduced-arrange display, spectator outline and eyewitness based diminished request controller plan

(5) Design of lessened request direct useful onlooker for huge scale discrete-time straight control frameworks.

First,using the modular approach, a recognizable proof of overwhelming and non-prevailing conditions of straight discrete-time frameworks is done. This strategy is utilized for acquiring the decreased request show for a given substantial scale direct discrete-time control framework. The lessened request show is utilized for control applications like decreased request controller, diminished request onlooker and spectator based diminished request controller for the direct control framework. Lyapunov steadiness hypothesis is the technique utilized for building up the decreased request controllers and onlookers for the substantial scale discrete-time straight systems.Reduced arrange control is utilized to balance out the framework and spectators are utilized to decide the framework is noticeable or not. Determine a Separation central which sets up that the spectator based decreased request controller without a doubt balances out the overwhelming condition of the given direct control framework and furthermore shows that the onlooker shafts and the shut circle controller posts can be isolated.

LITERATURE SURVEY:

[1]Aldeen and Trinh (1999)Reduced-order linear functional observer for linear system, which bases the observer order on the ratio of independent output measurements to independentinput estimations. This strategy is exceptionally helpful for high request frameworks having much more number of yields than inputs. It is demonstrated that the spectator asymptotically focalizes to any number of direct useful when some minor conditions are met.

[2] Parmer and Mukherjee (2007) Reduced request displaying of direct unique frameworks utilizing molecule swarm streamlined eigen range analysis.It utilizes the consolidated focal points of the eigen range examination and the mistake minimization by molecule swarm improvement method. Shaft centroid and framework solidness of both unique and lessened request frameworks stay same in this strategy to decide the poles,whereas zeros are combined by limiting the basic square mistake in the middle of the transient reactions of unique and decreased request models utilizing molecule swarm improvement system, relating to a unit step input.

[3] Chen, Chang and Han(1979)The strength condition technique is one of the mainstream strategies for the lessening of exchange elements of high-arrange direct control frameworks. This technique jam security in the diminished request display if the first high-arrange framework is steady. Some fascinating variations of the solidness condition strategy have likewise been acquired.**PROBLEM DEFINTION:**

The analysis and design of practical control systems become very complex, when the order of system increases. Therefore, to analyze such large scale linear control systems, it is necessary to reduce it to a lower order system, which is a sufficient representation of the higher order system. In recent decades, much effort has been made in the control systems literature for the model order reduction of large scale linear control systems.

Some important reasons for using reduced order models of high order linear control systems are the following:

(1) To have a better understanding of the system.
(2) To reduce computational complexity.
(3) To reduce hardware complexity.
(4) To make feasible controller design

In the analysis and design of complex systems, it is often necessary to simplify a high order system. The use of a reduced order model makes it easier to implement analysis, simulations and control system designs.

PROPOSED APPROACH:

1. **Identification of Dominant and Non-Dominant States of Linear Discrete-Time Control System**

Step 1. Calculate $\Omega_{ik}=\frac{Mik}{1-\lambda k}\sum_{j=1}^{m}\gamma kj$ for i,k= 1,2,…,n.This yields an n×nmatrix Ω, whose (i,k)th entry gives the participation measure of the kth eigenvalueλ_k in the ith state x_i.

Step 2. Calculate $\Theta_k=\sum_{i=1}^{n}\Omega ik$, for 1,2,.,n, where Θ_k represents the dominance of the kth eigenvalue $\lambda_k$in all the states.

Step 3. Sort the values of Θ_k in the order of dominance, starting from the most dominant to the least dominant. Accordingly, sort the associated eigenvalues in the order of dominance. Also, reorder the columns of the modal matrix M so that $z_1,z_2,\ldots,z_n$correspond to the renamed eigenvalues $\lambda_1,\lambda_2,\ldots,\lambda_n$in the order of dominance.

Step 4. Calculate $\varphi_{ik}=\left|\frac{\Omega ik}{k}\right|\times 100$for all values of i and k,whereφ_{ik}gives the relative dominance of the kth eigenvalue λ_k in the ith state $x^{i.}$

Step 5. Calculate $\psi_{il}=\sum_{k=1}^{l}\psi ik$ for all values of i and l.

Step 6. Finally, test the condition

$\psi_{il}=\varphi_{ik}\geq \varepsilon_i$for i=1,2,…,n,

for values of l starting from 1= l to l = n and where $\varepsilon_i$is chosen arbitrarily between 80% and 100% according to the degree of approximation required.Based on above condition,the first l eigenvalues are the dominant eigenvalues. The rest are non-dominant eigenvalues.Correspondingly, the first l states in the new coordinates viz. $z_1,z_2,\ldots,z_l$are the dominant states, while the rest are non-dominant states. Also, the first l states in the original coordinates viz.$z_1,z_2,\ldots,z_l$are the dominant states, while the rest are non-dominant states.

2. REDUCED ORDER MODEL DESIGN FOR LINEAR DISCRETE-TIME SYSTEMS

Step1: Using dominant and non-dominant state, then the linear system becomes

$$\begin{bmatrix} xs(k+1) \\ xf(k+1) \end{bmatrix}=\begin{bmatrix} Ass & Asf \\ Afs & Aff \end{bmatrix}\begin{bmatrix} xs(k) \\ xf(k) \end{bmatrix}+\begin{bmatrix} Bs \\ Bf \end{bmatrix}u(k)$$

$$y(k)=[Cs \;\; Cf]\begin{bmatrix} xs\,(k) \\ xf\,(k) \end{bmatrix}$$

Step2:the plant equation can be written as

$x_s(k+1)=A_{ss}x_s(k)+A_{sf}x_f(k)+B_su(k)$

$x_f(k+1)=A_{fs}x_s(k) + A_{ff}x_f(k) + B_fu(k)$

$y(k)=C_sx_s(k)+C_fx_f(k)$

Step3: Find a non-singular (modal) matrix P consisting of n linearly independent eigenvectors of A such that

$P^{-1}AP=\wedge$

Step4: New set of coordinates on state space are

$\xi=P^{-1}x$

Step5: In the new coordinates the plant becomes

$$\begin{bmatrix} \xi s(k+1) \\ \xi f(k+1) \end{bmatrix}=\begin{bmatrix} \wedge s & 0 \\ 0 & \wedge f \end{bmatrix}\begin{bmatrix} \xi s(k) \\ \xi f(k) \end{bmatrix}+P^{-1}Bu(k)$$

$$y(k)=CP\begin{bmatrix} \xi s\,(k) \\ \xi f\,(k) \end{bmatrix}$$

Step5: Next, the following assumptions were made:

(H1) As $k\rightarrow\infty$, $\xi_f\,(k+1)\approx\xi_f(k)$ i.e.$\xi_f$takes a constant value in the steady-state.

(H2) The matrix $I-\wedge_f$is invertible.

For large values of k,

$\xi_f(k)\approx\wedge_f\xi_f(k) + \acute{\Gamma}_fu(k)$

i.e.

$\xi_f(k)\approx(I-\wedge_f)^{-1}\acute{\Gamma}_fu(k)$

Step6:we obtain the reduced-order model of the linear plant in the ξ coordinates as

$\xi_s(k+1)=\wedge_s\xi_s\,(k)+\acute{\Gamma}_su(k)$

$y(k)=\psi_s\xi_s\,(k) + \psi_f(I-\wedge_f)^{-1}\acute{\Gamma}_fu(k)$

Step 7:the reduced-order model of the linear plant in the xcoordinates is obtained under the assumption (H3) :The matrix Q_{ff} is invertible

$x_s(k+1)=A_s^*x_s(k) + B_s^*u(k)$

$y(k) = C_S^* x_s(k) + D_S^* u(k)$

3. REDUCED ORDER CONTROLLER DESIGN FOR LARGE SCALE LINEAR DISCRETE-TIME CONTROL SYSTEMS

For the stabilization of large scale linear discrete-time control systems design the reduced order controllers using dominant state $x_s$of the system.

Step 1: find a state feedback control law that uses dominant state of linear system,it stabilize the system.

$$u(k)= -[K_S^* \quad 0]\begin{bmatrix} xs(k) \\ xf(k) \end{bmatrix}=-K_S^* x_s(k)$$

Step 2: Closed-loop system governed by the equations

$x_s(k+1)=(A_S^* - B_S^* K_S^*)x_s(k)$

$x_f(k+1) =(A_f^* - B_f^* K_f^*)x_s(k)$

Step 3: From the above equation, it follows that

$x_s(k)=(A_S^* - B_S^* K_S^*)x_s(0)$

feedback control law stabilizes the system if and only if it stabilizes the reduced-order linear system.

Step 4:if the system pair $(A_S^*, B_S^*)$is controllable, then the eigenvalues of the closedloop system matrix $A_S^* - B_S^* K_S^*$ can be arbitrarily placed in the complex plane.

Step 5: Find a gain matrix F_S^* such that the closed-loop system matrix $A_S^* - B_S^* K_S^*$ is convergent.

REDUCED ORDER OBSERVER DESIGN FOR LARGE SCALE LINEAR DISCRETE-TIME CONTROL SYSTEMS

Step1:Define the candidate observer to estimate the dominant state of the system.

$Z_s(k+1)=A_S^* z_s(k)+ B_S^* u(k)+K_S^* [y(k)-C_S^* z_s(k)- D_S^* u(k)]$

Step2: Define estimation error as $e = z_s - x_s$

Then $e(k) \to 0$ exponentially as $k \to 8$ if and only if the matrixK_S^* is such that $E=A_S^* - K_S^* C_S^*$ is convergent.

Step3: Then always construct an exponential observer having any desired speed of convergence if (C_S^*, A_S^*) is observable.

4. OBSERVER BASED REDUCED ORDER CONTROLLER DESIGN FOR LINEAR DISCRETE-TIME CONTROL SYSTEMS

Using Separation Principle, observer-based reduced-order controller is derived,itstabilizes the dominant state of the given linear control system and also demonstrates that the observer poles and the closed-loop controller poles can be separated.

Separation Principle

Step1: Under the assumption,the reduced order linear system is

$x_s(k+1) =A_S^* x_s(k)+B_S^* u(k)$

$y(k) =C_S^* x_s(k)+D_S^* u(k)$

Step2: Suppose that there exist matrices F_S^* and K_S^* such that $A_S^* - B_S^* F_S^*$ and $A_S^* - K_S^* C_S^*$ are both convergent matrices.

Step3: For the reduced order linear system, suppose that the pair (C_S^*, A_S^*) iscompletely observable. Then the system has a global exponential observer defined by

$Z_s(k+1)=A_S^* z_s(k)+ B_S^* u(k)+K_S^* [y(k)-C_S^* z_s(k)- D_S^* u(k)]$

Step4: Then the observer poles and the closed-loop controller poles are separated and the observer-based control law

$u(k)=-F_S^* z_s(k)$

alsostabilizes the dominant state x_s of the large-scale control system (8.27).

5. REDUCED ORDER LINEAR FUNCTIONAL OBSERVER DESIGN FOR LARGE SCALE LINEAR

DISCRETE-TIME CONTROL SYSTEMS

Step1: Using Ackermann's formula, find a state feedback control law

$$u(k) = Fx(k)$$

so that the closed-loop system matrix A+BF is convergent.

Step2: Find the order qof the reduced-order linear functional observer.

Step3: Choose the (m×q) elements of the matrix K arbitrarily.

Step4: Choose arbitrarily a stable (i.e. convergent) matrix . E For simplicity, choose E as adiagonal matrix consisting of stable eigenvalues $\lambda_1,\lambda_2,\ldots,\lambda_q$ lying inside the open unit disc of the complex plane, i.e. in the region

$\{\lambda \in C:|\lambda|<1\}$

Step5: Solve the augmented system for a matrix T

$$\begin{bmatrix}\Omega\\ \psi\end{bmatrix} t = \begin{bmatrix} f\\ 0\end{bmatrix}$$

Step6: The feedback gain matrix F=KT+WCis partitioned in to two parts

$$WC_1=(F-KT)\begin{bmatrix} Ip\\ 0\end{bmatrix}$$

And

$$(F-KT)\begin{bmatrix} 0\\ In-p\end{bmatrix}=0$$

From this, find the matrix W.

Step7: Find the matrix G for the equation

$G[C_1 0]=TA-ET$

Step8: Find H by the formula H= TB

Step9: As a result of Steps (1)-(8), obtain a reduced-order linear functional observer given by the formula

$$z(k+1) = Ez(k)+Hu(k)+Gy(k)$$

and a feedback controller by

$$u(k)=Fx(k)=K\dot{\eta}(k) + Wy(k)$$

In implementing the design algorithm in practice, above equation requires knowledge of the state functional

$$\dot{\eta}(k)=Tx(k)$$

RESULTS

The impulse responses of the reduced order plant are Plotted in Figure 1.1. The step responses of the reducedorder plant for the reduced order model are plotted in Figure 1.2.

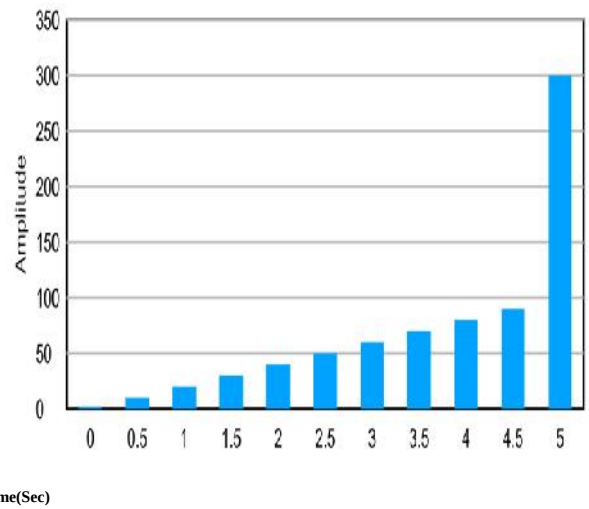

Fig.1.a Step response of reduced order plant

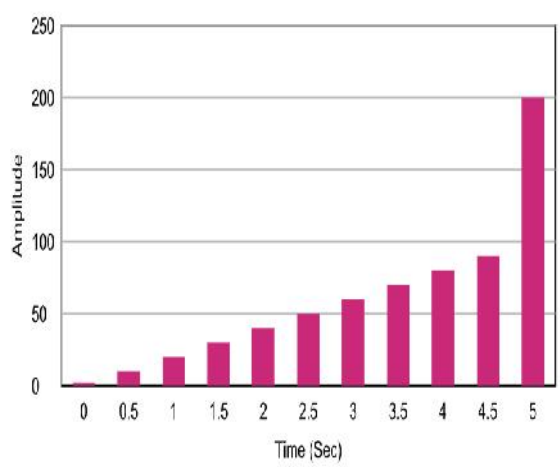

Fig1.b An Impulse response of reduced order plant

CONCLUSION

We initially recognize overwhelming and non-prevailing conditions of a direct framework utilizing modular approach.Develop a decreased request demonstrate for vast scale straight frameworks utilizing predominant and non-prevailing conditions of a framework. Utilizing lessened model we infer a condition for diminished request controllers and decreased request observersto balance out the framework. Utilizing this decreased request straight model, adequate conditions are inferred for the plan of onlooker based diminished request controllers. A division rule has been set up in this paper which shows that the eyewitness posts and controller shafts can be isolated and subsequently the post position issue and onlooker configuration are autonomous of each other. Vital and adequate conditions have been inferred for the development of lessened request direct utilitarian eyewitness for discrete-time straight control frameworks. A partition standard for usage of a spectator based input adjustment plot has been set up.

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